

# CHAPTER IV

## Climb Performance

An important quality of an airplane is its ability to gain altitude or climb. The rate of climb depends on engine power available which decreases with altitude and on power required, which is also a function of altitude. Therefore, rate-of-climb is greatly influenced by altitude. It decreases from a maximum value at sea level to zero at some value of altitude termed the ceiling.

### A. Rate-of-Climb

In order to establish the factors affecting rate-of-climb, consider the forces acting on an airplane in climbing configuration (Fig. 4-1). Summing the forces acting along the remote velocity vector

$$\Sigma F = T - D - W \sin \gamma$$

(equation 4-1)

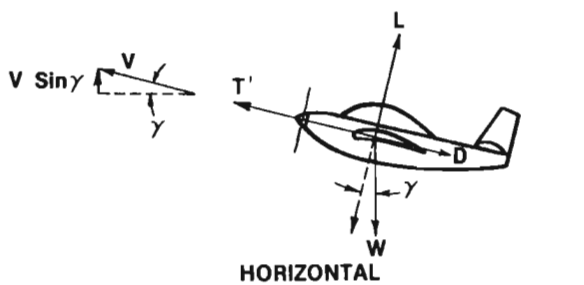


Fig. 4-1 Airplane in climb configuration

where  $\gamma$  is the angle between  $V$  (the airplane's flight path) and the horizontal plane. If the aircraft is in a steady level climb, no acceleration and hence, no net force, is present. Thus, the above expression may be equated to zero. Solving for  $\sin \gamma$  and multiplying both sides of the equation by  $V$  gives,

$$V \sin \gamma = \frac{(T - D)V}{W}$$

(equation 4-2)

$V \sin \gamma$ , however, is the vertical component of velocity or rate-of-climb. The symbol R/C is commonly used for rate-of-climb. Hence,

$$R/C = \frac{(T - D)V}{W}$$

(equation 4-3)

But  $TV$  is equal to power available and  $DV$  is equal to power required. So R/C, in terms of power, becomes

$$R/C = \frac{P_a - P_r}{W}$$

(equation 4-4)

Rate-of-climb is usually expressed in ft/min., so the right-hand side of this equation must be multiplied by 33,000 ft#/min/HP if power is expressed in horsepower.

Equation 4-4 shows that the rate-of-climb is directly proportional to the excess power available over that required for steady level flight. The maximum power available, then, which is the power available at full throttle, determines the maximum R/C.

A typical plot of both thrust power required and thrust power available curves for a given altitude is seen in Fig. 4-2. The power available as shown is the *maximum* available. By reducing the power settings, power available can be adjusted anywhere between the maximum value and zero. Power required, on the other hand, is a fixed curve representing the drag in terms of power. Notice that the  $\Delta P$  increment indicated on Fig. 4-2 is the maximum value for that altitude and, therefore, is located at the velocity for best climb.

Also notice on Fig. 4-2 that there is some point on the right where the curves cross. This point represents the maximum velocity at which steady level flight can be maintained. Beyond

that velocity  $P_r$  is greater than  $P_a$  and thus a negative value of R/C would result. Negative rate-of-climb is sometimes referred to as rate-of-sink (R/S). Remember that a rate-of-sink can be established at any airspeed by reducing  $P_a$  below  $P_r$  at that particular velocity. Maximum R/S results when  $P_a$  is reduced to zero.

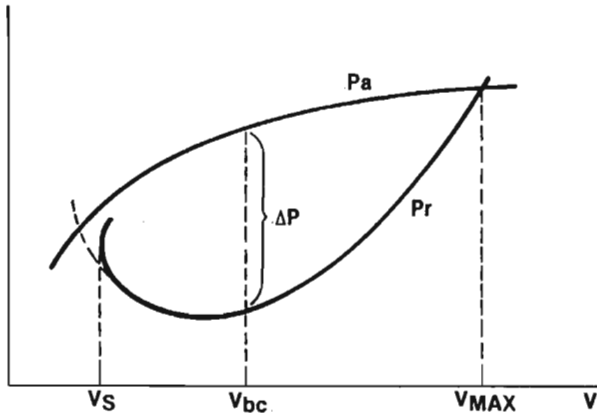


Fig. 4-2 Power available and required

There is also, theoretically, a point on the lower end of the graph where the curves cross, which could be called minimum velocity. This represents a velocity below which sufficient power is not available to maintain steady level flight because of higher induced power required. However, this point is usually of little significance since the actual drag increases rapidly near stall velocity and causes the  $P_r$  curve to curl up as indicated by the solid line portion. Thus, the stall velocity, usually higher than  $V_{min}$ , is normally the lower limit on velocity.

Since rate-of-climb is quite variable with altitude, velocity, and power setting, only maximum values at various altitudes are considered of significance. It is easy to realize that any value below maximum can be obtained by reducing power or varying velocity. Note that R/C decreases from maximum with either an increase or decrease in velocity from best climb speed.

### B. Ceiling

The procedure in flight testing rate-of-climb is to measure R/C at various velocities for a number of different altitudes. If the maximum R/C at each altitude is plotted versus altitude, a curve similar to that in Fig. 4-3 will result. The curve is usually linear but may have some slight

variations. If possible, it is desirable to run tests all the way from sea level to the altitude where R/C goes to zero. If this procedure is not possible or practical, an extrapolation can be made to carry both ends of the curve to its limits.

The maximum altitude where R/C becomes zero is called the *absolute ceiling*. Since this altitude is impractical to reach, a more meaningful limit on altitude is termed the *service ceiling*. Service ceiling is defined as the altitude where the R/C drops off to 100 ft/min. Extrapolating the lower end of the curve to zero altitude, of course, gives the value of R/C at sea level ( $R/C)_0$ .

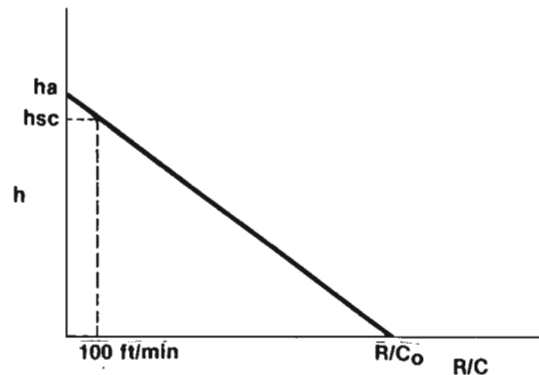


Fig. 4-3 Rate-of-climb vs altitude

### C. Flight Test Measurements

Flight testing for rate-of-climb usually consists of flying what is known as a "sawtooth climb" pattern. This is a series of climbs and descents through a certain nominal altitude. The time to climb a certain increment of altitude through this average altitude is recorded. A number of runs are made through the same altitude so that an average value can be obtained. In this way error due to the presence of vertical air currents is minimized. As in all flight testing, corrections must be made to the measured data to correct for non-standard weight and atmosphere.

Let us first determine a correction for non-standard atmosphere. Consider a column of air as in Fig. 4-4 with a height of  $\Delta h$ , the distance of our climb. Summing vertical forces on this column yields the equation,

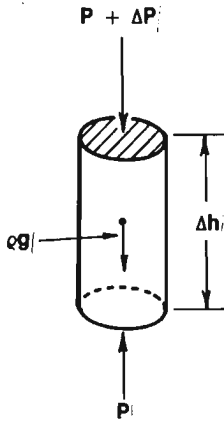


Fig. 4-4 Forces on an elemental column of air

$$P + \Delta P + \rho g \Delta h = P$$

(equation 4-5)

$$\Delta P = -\rho g \Delta h = -\frac{Pg}{RT} \Delta h$$

(equation 4-6)

Since the  $\Delta h$  as indicated by the altimeter is measured by a  $\Delta P$  based on standard rate of temperature lapse, the  $T$  in equation 4-6 is standard temperature ( $T_s$ ) and  $\Delta h$  is the measured change in altitude.

$$\Delta P = -\frac{Pg}{RT_s} \Delta h_m$$

This same amount of pressure change would indicate the actual  $\Delta h$  if the altimeter were based on the actual temperature. Thus, also,

$$\Delta P = -\frac{Pg}{RT_{act}} \Delta h_{act}$$

(equation 4-8)

Equating these two expressions for  $\Delta P$  yields the relation,

$$\frac{\Delta h_m}{T_s} = \frac{\Delta h_{act}}{T_{act}}$$

(equation 4-9)

Rearranging,

$$\begin{aligned} \Delta h_{act} &= \Delta h_m \frac{T_{act}}{T_s} = \Delta h_m \frac{T_{act}/T_o}{T_s/T_o} \\ &= \Delta h_m \frac{T_{act}/T_o}{(1 - .689 \times 10^{-5} h_p)} \end{aligned}$$

(equation 4-10)

Equation 4-10 provides an expression for correcting measured change in height to an actual change by inserting the appropriate actual OAT.

Since the test will probably not be flown at exactly standard weight a further correction must be made to account for the weight difference. To determine the effect of weight on R/C, equation 4-3 can be differentiated with respect to weight.

$$R/C = \frac{TV}{W} - \frac{DV}{W}$$

$$\frac{dR/C}{dW} = -\frac{TV}{W^2} + \frac{DV}{W^2} - \frac{V}{W} \left( \frac{dD}{dW} \right)$$

(equation 4-11)

The last term occurs because weight is an implicit function in the drag equation. From equation 3-9,

$$\frac{dD}{dW} = \frac{4W}{\pi \rho e b^2 V^2}$$

(equation 4-12)

Substituting this value into equation 4-11,

$$\frac{dR/C}{dW} = -\frac{1}{W} \left( \frac{TV - DV}{W} \right) - \frac{4}{\pi \rho e b^2 V}$$

(equation 4-13)

The term in the brackets turns out to be R/C, so the equation can be rewritten,

$$\frac{dR/C}{dW} = -\frac{1}{W} (R/C) - \frac{4}{\pi \rho e b^2 V}$$

(equation 4-14)